A note on neutrino radiation fields with positive energy density

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Letters to the Editor

A note on neutrino–gravitational fields with positive energy density

Abstract. Neutrino–gravitational fields with positive energy density are considered. A special case of these fields appeared to introduce some complications. It is shown here that this case may be removed by a simple transformation.

Neutrino fields with positive energy density have recently been considered in the general theory of relativity by Griffiths and Newing (1971, to be referred to as I), Wainwright (1971), Audretsch (1971) and Collinson (1971). In this letter we will be using the notation and techniques described in I.

It was shown in I that the energy momentum tensor of neutrino–gravitational fields with positive energy density may be written in the form

$$E_{\mu\nu} = -2Al_\mu l_\nu + 2\omega(l_\mu n_\nu + n_\mu l_\nu + m_\mu \bar{m}_\nu + \bar{m}_\mu m_\nu) + 2\mathrm{i}\bar{\sigma} m_\mu m_\nu - 2\mathrm{i}\sigma \bar{m}_\mu \bar{m}_\nu \quad (1)$$

where $A \leqslant 0$ and $\omega \geqslant \frac{1}{2}|\sigma|$. An exceptional case was also referred to which also possesses positive energy density, the energy momentum tensor being given by

$$E_{\mu\nu} = -2Al_\mu l_\nu + B(l_\mu m_\nu + m_\mu l_\nu) + \bar{B}(l_\mu \bar{m}_\nu + \bar{m}_\mu l_\nu) + |\sigma|(l_\mu n_\nu + n_\mu l_\nu + m_\mu \bar{m}_\nu + \bar{m}_\mu m_\nu) + 2\mathrm{i}\bar{\sigma} m_\mu m_\nu - 2\mathrm{i}\sigma \bar{m}_\mu \bar{m}_\nu \quad (2)$$

where $A < 0$, $\sigma B^2 = \mathrm{i}|\sigma||B|^2$ and $|B|^2 \leqslant -4A|\sigma|$. We will now show that this does not form a separate case but that we may always put $B = 0$, the energy momentum tensor thus becoming of the form (1), for which $\omega = \frac{1}{2}|\sigma|$.

It was shown in I that we are free to make an arbitrary ψ-transformation on the null tetrad. Under this transformation

$$B \to B' = B + 4\omega\bar{\psi} + 2\mathrm{i}\bar{\sigma}\psi \quad (3)$$

and hence ψ may be chosen to make B' zero unless $\omega = 0 = \sigma$ or $\omega = \frac{1}{2}|\sigma|$. The exceptional case referred to is the case when $\omega = \frac{1}{2}|\sigma|$. By considering the positive energy condition we find that an energy momentum tensor of the form (2) will have positive energy density provided $A < 0$, $\sigma B^2 = \mathrm{i}|\sigma||B|^2$ and $|B|^2 \leqslant -4A|\sigma|$. Now the condition $\sigma B^2 = \mathrm{i}|\sigma||B|^2$ is the condition on (3) for the existence of a ψ which will make $B' = 0$ when $\omega = \frac{1}{2}|\sigma|$. This can be seen by putting $B = b\exp(\mathrm{i}\beta)$, $\sigma = 2\omega\exp(\mathrm{i}\theta)$ and $\psi = \Psi\exp(\mathrm{i}\eta)$. This condition then becomes

$$\theta + 2\beta = \frac{\pi}{2}$$

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and (3) becomes

$$\begin{aligned} B' &= b\exp(i\beta) + 4\omega\Psi\left[\exp(-i\eta) + \exp\left\{i\left(\eta - \theta + \frac{\pi}{2}\right)\right\}\right] \\ &= b\exp(i\beta) + 4\omega\Psi[\exp(-i\eta) + \exp\{i(\eta + 2\beta)\}] \\ &= \exp(i\beta)\{b + 4\omega\Psi\cos(\eta + \beta)\}. \end{aligned}$$

It can be seen from this equation that it is always possible to choose a ψ such that $B' = 0$. For example, we make take

$$\psi = -\frac{b}{8\omega}\exp(-i\beta).$$

This reduces (2) to the form (1) for which $\omega = \frac{1}{2}|\sigma|$.

It has thus been established that the energy momentum tensor for neutrino fields with positive energy density may always be written in the form (1). This is in agreement with results established by Wainwright (1971).

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Shape of a self-avoiding walk or polymer chain

Abstract. If $p_n(\mathbf{r})$ is the probability that a self-avoiding walk of n steps reaches a distance $\mathbf{r}$ from the origin, then it is shown, for large n and $r \gg R_n$, that

$$p_n(\mathbf{r}) \sim R_n^{-d}(r/R_n)^t \exp\{-(r/R_n)^{1/(1-\nu)}\}$$

where R_n is a scaling length which varies as n^ν, and d is the dimensionality. Furthermore, the index t is related to d, ν, and a further index γ which describes the asymptotic behaviour of the total number of self-avoiding walks.

We have also shown, on the assumption that $p_n(\mathbf{r}) \sim R_n^{-d}(r/R_n)^g$ for large n and $r \ll R_n$ that the index g can be related to d, ν, γ, and an index α which describes the asymptotic behaviour of the total number of self-avoiding walks which return to the origin.

A self-avoiding walk on a lattice is a random walk such that no lattice site is visited more than once in the walk. Such walks are of interest as models of polymer chains in which 'excluded volume' effects are important. Furthermore such walks are connected with certain properties of the Ising model (Domb 1969, Fisher 1966) so that a study of their properties may have application in the more general problem of second-order phase transitions. In this note, we use the analogy between the